NAME 323: RESISTANCE & PROPULSION OF SHIPS

3.00 Credit , 3.0 Hours/week

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4.2.5. Frictional Resistance Coefficient

As was mentioned in Section 4.1 the frictional resistance R_F is the component of resistance obtained by integrating the tangential stresses over the wetted surface of the ship in the direction of motion.

The specific frictional resistance or drag coefficient C_F is then defined by

$$C_F = \frac{R_F}{\frac{1}{2}\rho V^2 S} \qquad (4.2.12)$$

where ρ is the mass density, V is the speed of the ship, and S is the wetted surface of the ship.

On the other hand, if C_F is known at the speed V in question, the frictional resistance can be calculated by

$$R_F = C_F(\tfrac{1}{2}\rho V^2)S \qquad (4.2.13)$$

4.3 Flat Plate Friction Formulae

The level and slope of the skin friction line is fundamental to the extrapolation of resistance data from model to ship, as discussed in Section 4.1. The following sections outline the principal skin friction lines employed in ship resistance work.

4.3.1 Froude Experiments

The first systematic experiments to determine frictional resistance in water of thin flat planks were carried out in the late 1860s by W. Froude. He used planks 19 in deep, 3/16 in thick and lengths of 2 to 50 ft, coated in different ways [4.1, 4.2] A mechanical dynamometer was used to measure the total model resistance, Barnaby [4.11], using speeds from 0 to 800 ft/min (4 m/s).

Froude found that he could express the results in the empirical formula

$$R = f \cdot S \cdot V^n. \tag{4.6}$$

The coefficient f and index n were found to vary for both type and length of surface. The original findings are summarised as follows:

1. The coefficient f decreased with increasing plank length, with the exception of very short lengths.
2. The index n is appreciably less than 2 with the exception of rough surfaces when it approaches 2
3. The degree of roughness of the surface has a marked influence on the magnitude of f.

Froude summarised his values for f and n for varnish, paraffin wax, fine sand and coarse sand for plank lengths up to 50 ft (for >50 ft Froude suggested using f for 49–50 ft).

R. E. Froude (son of W. Froude) re-examined the results obtained by his father and, together with data from other experiments, considered that the results of planks having surfaces corresponding to those of clean ship hulls or to paraffin wax models could be expressed as the following:

$$R_F = f \cdot S \cdot V^{1.825}, \tag{4.7}$$

with associated table of f values, see Table 4.1.

If Froude's data are plotted on a Reynolds Number base, then the results appear as follows:

$$R = f \cdot S \cdot V^{1.825},$$

$$C_F = R/\tfrac{1}{2}\rho S V^2 = 2 \cdot f \cdot V^{-0.175}/\rho$$

$$= 2 \cdot f \cdot V^{-0.175} \cdot Re^{-0.175}/\rho L^{-0.175},$$

then

$$C_F = f' \cdot Re^{-0.175},$$

Table 4.1. *R.E. Froude's skin friction f values*

Length (m)	f	Length (m)	f	Length (m)	f
2.0	1.966	11	1.589	40	1.464
2.5	1.913	12	1.577	45	1.459
3.0	1.867	13	1.566	50	1.454
3.5	1.826	14	1.556	60	1.447
4.0	1.791	15	1.547	70	1.441
4.5	1.761	16	1.539	80	1.437
5.0	1.736	17	1.532	90	1.432
5.5	1.715	18	1.526	100	1.428
6.0	1.696	19	1.520	120	1.421
6.5	1.681	20	1.515	140	1.415
7.0	1.667	22	1.506	160	1.410
7.5	1.654	24	1.499	180	1.404
8.0	1.643	26	1.492	200	1.399
8.5	1.632	28	1.487	250	1.389
9.0	1.622	30	1.482	300	1.380
9.5	1.613	35	1.472	350	1.373
10.0	1.604				

where f' depends on length. According to the data, f' increases with length as seen in Figure 4.6 [4.12].

ships when the power by Froude is overestimated (by up to 15%). Froude f values are listed in Table 4.1, where L is waterline length (m) and units in Equation (4.7) are as follows: V is speed (m/s), S is wetted area (m^2) and R_F is frictional resistance (N).

A reasonable approximation (within 1.5%) to the table of f values is

$$f = 1.38 + 9.4/[8.8 + (L \times 3.28)] \text{ (}L\text{ in metres).} \qquad (4.8)$$

R. E. Froude also established the circular non-dimensional notation, [4.14], together with the use of 'O' values for the skin friction correction, see Sections 3.1 and 10.3.

4.3.2 Schoenherr Formula

In the early 1920s Von Karman deduced a friction law for flat plates based on a two-dimensional analysis of turbulent boundary layers. He produced a theoretical 'smooth turbulent' friction law of the following form:

$$1/\sqrt{C_F} = A + B\ \mathrm{Log}\,(Re \cdot C_F), \tag{4.9}$$

where A and B were two undetermined constants. Following the publication of this formula, Schoenherr replotted all the available experimental data from plank experiments both in air and water and attempted to determine the constants A and B to suit the available data, [4.15]. He determined the following formula:

$$1/\sqrt{C_F} = 4.13\log_{10}(Re \cdot C_F) \tag{4.10}$$

be judged in this light. The Schoenherr line was adopted by the American Towing Tank Conference (ATTC) in 1947. When using the Schoenherr line for model-ship extrapolation, it has been common practice to add a roughness allowance $\Delta C_F = 0.0004$ to the ship value, see Figure 4.6.

The Schoenherr formula is not very convenient to use since C_F is not explicitly defined for a given Re. In order to determine C_F for a given Re, it is necessary to assume a range of C_F, calculate the corresponding Re and then interpolate. Such iterations are, however, simple to carry out using a computer or spreadsheet. A reasonable fit to the Schoenherr line (within 1%) for preliminary power estimates is given in [4.17]

$$C_F = \frac{1}{(3.5 \log_{10} Re - 5.96)^2}. \tag{4.11}$$

4.3.3 The ITTC Formula

Several proposals for a more direct formula which approximates the Schoenherr values have been made. The Schoenherr formula (Equation (4.10)) can be expanded as follows:

$$1/\sqrt{C_F} = 4.13 \log_{10}(Re.C_F) = 4.13(\log_{10} Re + \log_{10} C_F). \tag{4.12}$$

C_F and $\log C_F$ vary comparatively slowly with Re as shown in Table 4.2. Thus, a formula of the form

$$1/\sqrt{C_F} = A(\log_{10} Re - B)$$

may not be an unreasonable approximation with B assumed as 2. The formula can then be rewritten as:

$$C_F = \frac{A'}{(\log_{10} Re - 2)^2}. \tag{4.13}$$

There are several variations of this formula type which are, essentially, approximations of the Schoenherr formula.

In 1957 the ITTC adopted one such formula for use as a 'correlation line' in powering calculations. It is termed the 'ITTC1957 model-ship correlation line'. This formula was based on a proposal by Hughes [4.4] for a two-dimensional line of the following form:

$$C_F = \frac{0.066}{(\log_{10} Re - 2.03)^2}. \tag{4.14}$$

The ITTC1957 formula incorporates some three-dimensional friction effects and is defined as:

$$C_F = \frac{0.075}{(\log_{10} Re - 2)^2}. \tag{4.15}$$

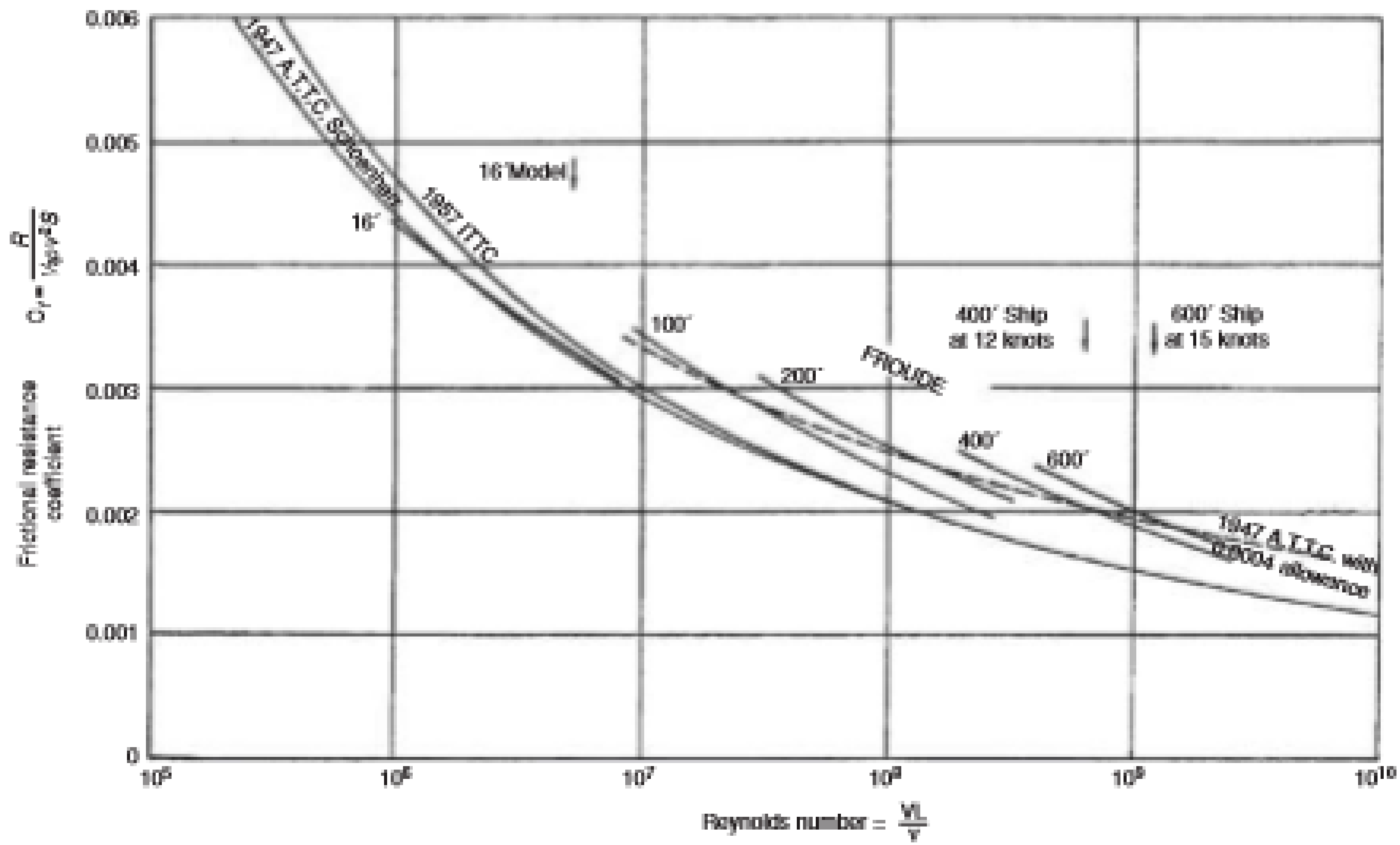

Figure 4.6. Comparison of different friction formulae.

4.1.2 Form Factor Approach: Hughes

Hughes proposed taking form effect into account in the extrapolation process. The basis of the approach is summarised as follows:

$$C_T = (1+k)C_F + C_W \tag{4.3}$$

or

$$C_T = C_V + C_W, \tag{4.4}$$

where

$$C_V = (1+k)C_F,$$

and $(1+k)$ is a form factor which depends on hull form, C_F is the skin friction coefficient based on flat plate results, C_V is a viscous coefficient taking account of both skin friction and viscous pressure resistance and C_W is the wave resistance coefficient. The method is shown schematically in Figure 4.2.

On the basis of Froude's law,

$$C_{Ws} = C_{Wm}$$

and

$$C_{Ts} = C_{Tm} - (1+k)(C_{Fm} - C_{Fs}). \tag{4.5}$$

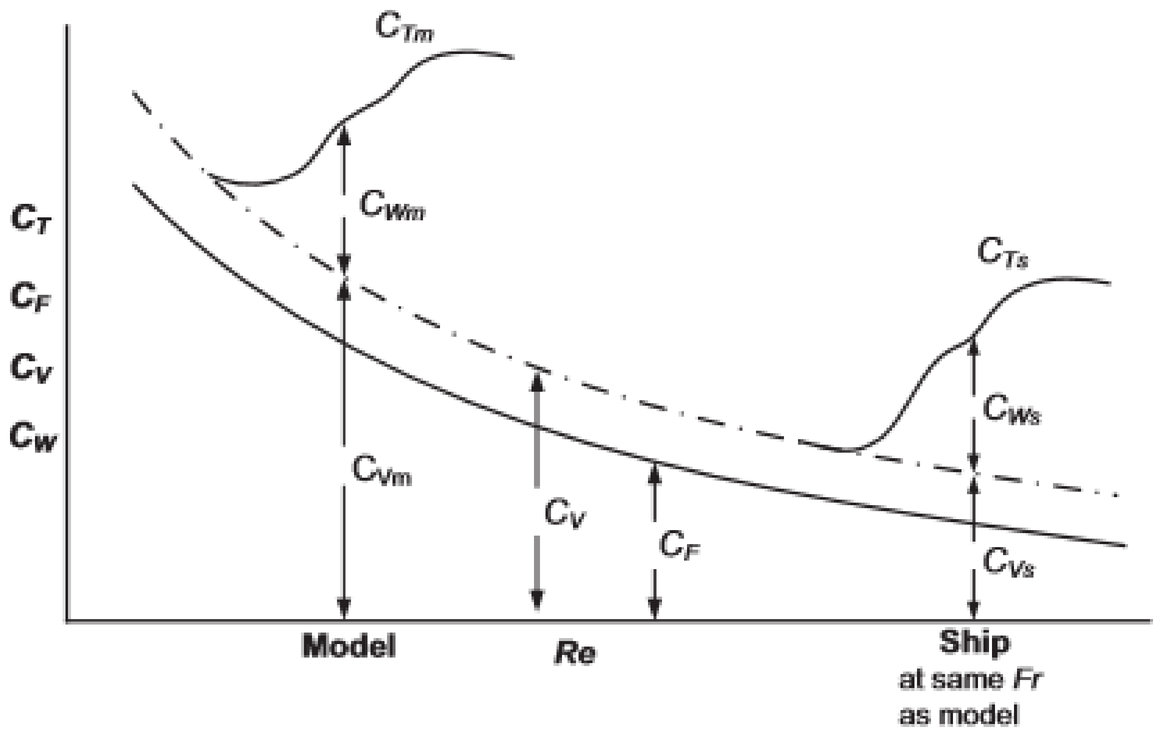

Figure 4.2. Model-ship extrapolation: form factor approach.

This method is recommended by ITTC and is the one adopted by most naval architects. A form factor approach may not be applied for some high-speed craft and for yachts.

The form factor $(1 + k)$ depends on the hull form and may be derived from low-speed tests when, at low Fr, wave resistance C_W tends to zero and $(1 + k) = C_{Tm}/C_{Fm}$. This and other methods of obtaining the form factor are described in Section 4.4.

4.4 Derivation of Form Factor $(1 + k)$

It is clear from Equation (4.5) that the size of the form factor has a direct influence on the model to ship extrapolation process and the size of the ship resistance estim-
resistance components, i.e. Re and Fr dependency. For example, when extrapolating model resistance to full scale, an increase in derived (or assumed) model $(1 + k)$ will result in a decrease in C_W and a decrease in the estimated full-scale resistance. Methods of estimating $(1+ k)$ include experimental, numerical and empirical.

4.4.1 Model Experiments

There are a number of model experiments that allow the form factor to be derived directly or indirectly. These are summarised as follows:

1. The model is tested at very low Fr until C_T runs parallel with C_F, Figure 4.9. In this case, C_W tends to zero and $(1 + k) = C_T/C_F$.
2. C_W is extrapolated back at low speeds. The procedure assumes that:

$$R_W \propto V^6 \quad \text{or} \quad C_W \propto R_W/V^2 \propto V^4$$

that is

$$C_W \propto Fr^4, \quad \text{or} \quad C_W = A Fr^4,$$

where A is a constant. Hence, from two measurements of C_T at *relatively* low speeds, and using $C_T = (1 + k)\, C_F + A\, Fr^4$, $(1 + k)$ can be found. Speeds as low as $Fr = 0.1\sim0.2$ are necessary for this method and a problem exists in that it is generally difficult to achieve accurate resistance measurements at such low speeds.

The methods described are attributable to Hughes. Prohaska [4.23] uses a similar technique but applies more data points to the equation as follows:

$$C_T/C_F = (1+k) + AFr^4/C_F, \qquad (4.18)$$

where the intercept is $(1 + k)$, and the slope is A, Figure 4.10.

For full form vessels the points may not plot on a straight line and a power of Fr between 4 and 6 may be more appropriate.

A later ITTC recommendation as a modification to Prohaska is

$$C_T/C_F = (1+k) + A\,Fr^n/C_F, \qquad (4.19)$$

where n, A and k are derived from a least-squares approximation.

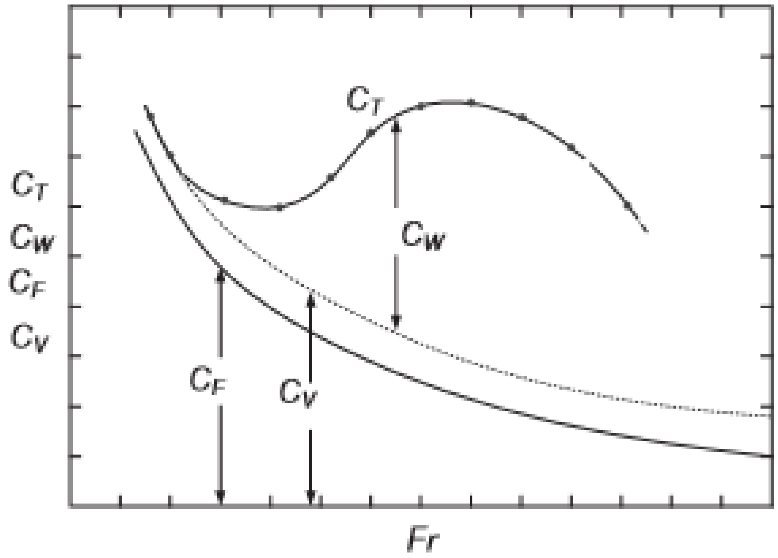

Figure 4.9. Resistance components.

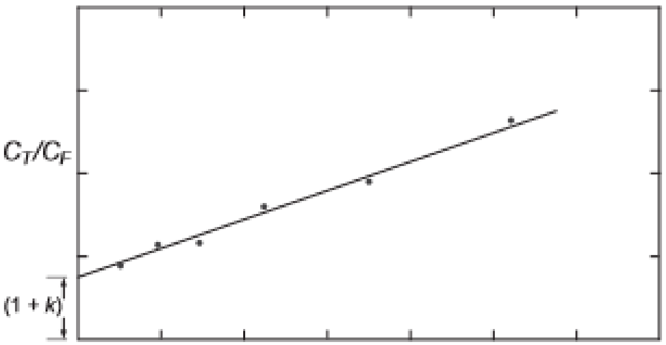

Figure 4.10. Prohaska plot.

3. $(1+k)$ from direct physical measurement of resistance components:

$$C_T = (1+k)C_F + C_W$$
$$= C_V + C_W.$$

(a) Measurement of total viscous drag, C_V (e.g. from a wake traverse; see Chapter 7):

$$C_V = (1+k)\,C_F, \text{ and } (1+k) = C_V/C_F.$$

(b) Measurement of wave pattern drag, C_W (e.g. using wave probes, see Chapter 7):

$$(1+k)\,C_F = C_T - C_W, \text{ and } (1+k) = (C_T - C_W)\,/\,C_F.$$

Methods 3(a) and 3(b) are generally used for research purposes, rather than for routine testing, although measurement of wave pattern drag on a routine basis is a practical option. It should be noted that methods 3(a) and 3(b) allow the derivation of $(1+k)$ over the whole speed range and should indicate any likely changes in $(1+k)$ with speed.

4.4.3 Empirical Methods

Several investigators have developed empirical formulae for $(1 + k)$ based on model test results. The following are some examples which may be used for practical powering purposes.

Table 4.3. *C_{stern} parameter*

Afterbody form	C_{stern}
Pram with gondola	−25
V-shaped sections	−10
Normal section shape	0
U-shaped sections with Hogner stern	10

Watanabe:

$$k = -0.095 + 25.6\frac{C_B}{\left[\frac{L}{B}\right]^2 \sqrt{\frac{B}{T}}}. \tag{4.20}$$

Conn and Ferguson [4.9]:

$$k = 18.7\left[C_B\frac{B}{L}\right]^2. \tag{4.21}$$

Grigson [4.21], based on a slightly modified ITTC line:

$$k = 0.028 + 3.30\left[\frac{S}{L^2}\sqrt{C_B\frac{B}{L}}\right]. \tag{4.22}$$

Holtrop regression [4.25]:

$$(1+k) = 0.93 + 0.487118(1+0.011C_{\text{stern}}) \times (B/L)^{1.06806}(T/L)^{0.46106} \times (L_{WL}/L_R)^{0.121563}(L_{WL}^3/\nabla)^{0.36486} \times (1-C_P)^{-0.604247}. \quad (4.23)$$

If the length of run L_R is not known, it may be estimated using the following formula:

$$L_R = L_{WL}\left[1 - C_P + \frac{0.06C_P LCB}{(4C_P - 1)}\right], \quad (4.24)$$

where LCB is a percentage of L_{WL} forward of $0.5L_{WL}$. The stern shape parameter C_{stern} for different hull forms is shown in Table 4.3.

Wright [4.26]:

$$(1+k) = 2.480\ C_B^{0.1526}(B/T)^{0.0533}(B/L_{BP})^{0.3856}. \quad (4.25)$$

Couser *et al.* [4.27], suitable for round bilge monohulls and catamarans:

$$\text{Monohulls: } (1+k) = 2.76(L/\nabla^{1/3})^{-0.4}. \quad (4.26)$$

$$\text{Catamarans: } (1+\beta k) = 3.03(L/\nabla^{1/3})^{-0.40}. \quad (4.27)$$

For practical purposes, the form factor is assumed to remain constant over the speed range and between model and ship.