NAME 323: RESISTANCE & PROPULSION OF SHIPS

3.00 Credit , 3.0 Hours/week

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Statistical Method

In the preliminary stages of ship design, the resistance coefficient is estimated with approximate methods based on systematic series or statistical regressions to experimental data.

A systematic series is a family of ship hulls obtained from a systematic variation of one or more shape parameters. Usually, the changes are based on a parent form. The resistance of all the models that constitute a series is measured experimentally. This database allows the interpolation of the resistance coefficient for other shapes originated by parametric variations of the original shape.

5.4. USE OF STATISTICAL METHODS

5.4.1. Introduction

Doust (1962, 1964) was one of the first to demonstrate how statistical theory could be applied in ship design and power estimating. Using a computer, the method yields a regression equation that expresses ship resistance for a particular ship type in terms of certain basic form parameters at any required Froude number. Evaluation of this regression equation for specific combinations of form parameters provides corresponding estimates of resistance for the vessel under consideration, while minimization of the equation within the practical ranges of the form parameters gives an indication of where improvements in ship resistance can be made in specific cases.

5.4.2. Regression Analysis

In this section a statistical method to evaluate a series of results from towing tests with ship models of one particular ship type is given.

The procedure in a regression analysis can be as follows:

1. A series of model test results is established for the ship type in question. The number of experiments is called n.
2. The variation of the residuary resistance coefficient C_R as function of the ship form is wanted:

$$y = C_R = f(x_1, x_2, x_3, \ldots) \qquad (5.4.1)$$

3. The parameters x that can have an influence on C_R are studied. If there is a correlation between two of the parameters, one of them is rejected.

For a constant value of Froude number F_n the parameters in the analysis may be

L/B = length–breadth ratio

B/T = breadth–draught ratio

β = maximum sectional area coefficient

φ = horizontal prismatic coefficient ($\varphi = \delta/\beta$, where δ is the block coefficient and β the midship section coefficient)

LCB = longitudinal position of center of buoyancy (or the distance of CB abaft amidships and expressed as a percentage of L)

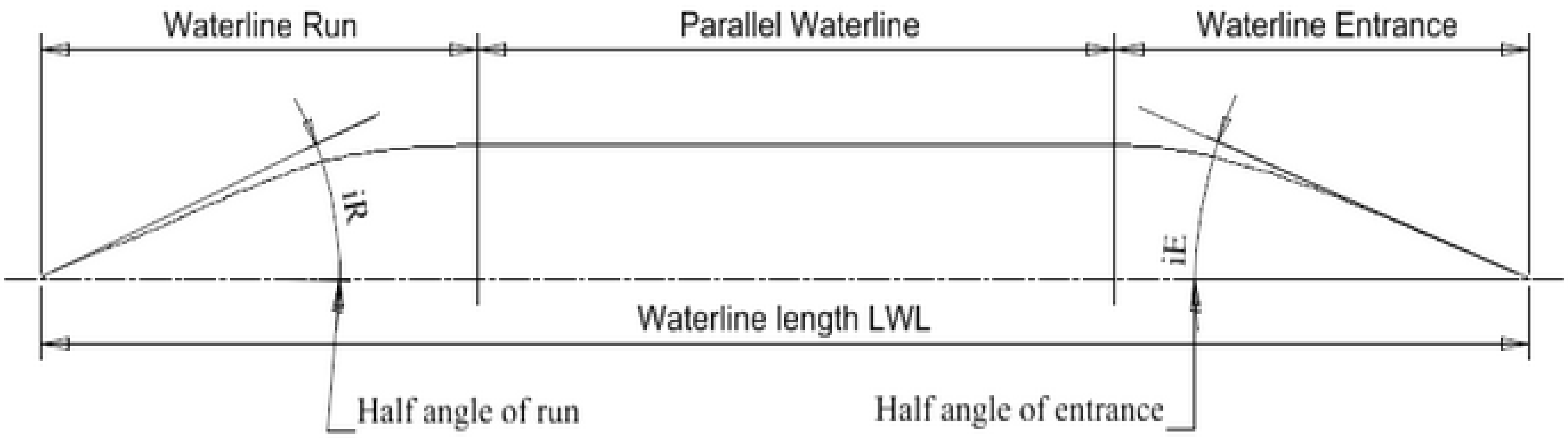
$\frac{1}{2}\alpha_e$ = half-angle of entrance measured on the floating waterline forward

$\frac{1}{2}\alpha_p$ = maximum angle of run at a station one-half of the underwater form

α_{BS} = maximum buttock slope at $B/4$ of the underwater form measured relative to floating waterline

The angle of the waterline at the bow in the horizontal plane neglecting local shape at stern is the *Angle of the Half angle of entrance* (i_E) [-] i.e. with respect to the centreplane - See Figure below.

The angle of the waterline at the stern in the horizontal plane neglecting local shape of stern frame is designated as *Half angle of run* (i_R) [-] i.e. with respect to the centreplane – See figure below.



4. All the basic parameters and the resistance coefficient have to be transformed into new variables, which range from −1 to +1. In this way the terms are all of the same order of magnitude, thus avoiding any problem with varying decimal points, while the relative importance of the various terms can be more easily assessed. The relation between the parameters and the new variables will then be

$$x = K_1(\text{parameter} - K_2) \tag{5.4.2}$$

5. The result from each individual experiment is arranged in an equation of the form:

$$y = b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + \cdots + b_mx_m + \cdots + b_px_p + e \tag{5.4.3}$$

where x_1 to x_m are the variables and x_{m+1} to x_p are new independent variables composed of products of combinations and powers of the variables x_1 to x_m. In the Eq. (5.4.3) e is the residual error in each observation and b_1 to b_p are the regression coefficients to be determined.

6. The equations from the individual experiments [Eq. (5.4.3)] are then collected in a system of equations (a mathematical model):

$$
\begin{aligned}
y^{(1)} &= b_1x_1^{(1)} + b_2x_2^{(1)} + b_3x_3^{(1)} \\
&\qquad + b_4x_4^{(1)} + \cdots + b_px_p^{(1)} + e^{(1)} \\
y^{(2)} &= b_1x_1^{(2)} + b_2x_2^{(2)} + b_3x_3^{(2)} \\
&\qquad + b_4x_4^{(2)} + \cdots + b_px_p^{(2)} + e^{(2)} \\
y^{(3)} &= b_1x_1^{(3)} + b_2x_2^{(3)} + b_3x_3^{(3)} \\
&\qquad + b_4x_4^{(3)} + \cdots + b_px_p^{(3)} + e^{(3)} \\
y^{(4)} &= b_1x_1^{(4)} + b_2x_2^{(4)} + b_3x_3^{(4)} \\
&\qquad + b_4x_4^{(4)} + \cdots + b_px_p^{(4)} + e^{(4)} \\
&\vdots \\
y^{(n)} &= b_1x_1^{(n)} + b_2x_2^{(n)} + b_3x_3^{(n)} \\
&\qquad + b_4x_4^{(n)} + \cdots + b_px_p^{(n)} + e^{(n)}
\end{aligned}
\tag{5.4.4}
$$

The summation in Eq. (5.4.5) and in the following equations is taken from $i = 1$ to $i = n$, where n is the number of individual experiments. By differentiation with respect to $b_1, b_2, b_3, \ldots, b_p$ and equating to zero a series of simultaneous equations is obtained:

$$b_1\Sigma x_1^2 + b_2\Sigma x_1x_2 + b_3\Sigma x_1x_3 + b_4\Sigma x_1x_4 + \ldots + b_p\Sigma x_1x_p = \Sigma x_1y$$

$$b_1\Sigma x_1x_2 + b_2\Sigma x_2^2 + b_3\Sigma x_2x_3 + b_4\Sigma x_2x_4 + \ldots + b_p\Sigma x_2x_p = \Sigma x_2y$$

$$b_1\Sigma x_1x_3 + b_2\Sigma x_2x_3 + b_3\Sigma x_3^2 + b_4\Sigma x_3x_4 + \ldots + b_p\Sigma x_3x_p = \Sigma x_3y \quad (5.4.6)$$

$$[N][B]=[D]$$

$$[N]^T[N][B]=[N]^T[D]$$

$$[B]=[[N]^T[N]]^{-1}[N]^T[D]$$

By using vector notation, the following is introduced:

$$x^{(i)} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_p \end{bmatrix}^{(i)} \tag{5.4.7}$$

which is a $(p \times 1)$ column vector;

$$\hat{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_p \end{bmatrix} \tag{5.4.8}$$

which is a $(p \times 1)$ column vector;

$$y^{(i)} = [y]^{(i)} \tag{5.4.9}$$

which is a (1×1) matrix.

For simplification the indices (i) will be omitted in the following equations: $\bar{x}'$ is the transpose of $\bar{x}$ and is a $(1 \times p)$ row vector. $\bar{x}\bar{x}'$ will then be a quadratic matrix $(p \times p)$. $(\bar{x} \cdot \bar{x}') \cdot \hat{b}$ will be a $(p \times 1)$ column vector $[(p \times p) \cdot (p \times 1) = (p \times 1)]$. Also $\bar{x} \cdot y$ will be a $(p \times 1)$ column vector, which in matrix notation means that

$$(\bar{x}\bar{x}')\hat{b} = \bar{x} \cdot y \qquad (5.4.10)$$

where $\hat{b}$ are the regression coefficients that have to be estimated. Then a summation from $i = 1$ to $i = n$ has to be carried out. The new system of equations will be similar to Eq. (5.4.10), which means:

$$\bar{A} \cdot \hat{b} = \bar{B} \qquad (p \times p)(p \times 1) = (p \times 1) \qquad (5.4.11)$$

or

$$\hat{b} = \bar{A}^{-1}\bar{B} \qquad (5.4.12)$$

where $\bar{A} = \bar{x}\bar{x}'$ and $\bar{B} = \Sigma\bar{x}y$. For each row or for each i from 1 to n in the system of equations (5.4.4) the following notation can be used:

$$y = \bar{x}' \cdot \hat{b} + \hat{e} \qquad (5.4.13)$$

or

$$\hat{e} = y - \bar{x}' \cdot \hat{b} \qquad (5.4.14)$$

Since e is a number

$$\hat{e} = \hat{e}' \quad (e \text{ transposed}) \qquad (5.4.15)$$

Therefore,

$$e^2 = \hat{e} \cdot \hat{e}' = (y - \bar{x}'\hat{b})(y - \bar{x}'\hat{b})'$$
$$(5.4.16)$$
$$= (y - \bar{x}'\hat{b})(y' - \hat{b}'\bar{x})$$

because $(\bar{x}')' = \bar{x}$. Performing the multiplication and remembering that every term in Eq. (5.4.16) is a (1×1) matrix, the following equation is obtained:

$$e^2 = yy' - 2\hat{b}'\bar{x}y + \hat{b}' \cdot \bar{x}\bar{x}' \cdot \hat{b} \qquad (5.4.17)$$

For each $i = 1, 2, \ldots, n, y$ is a number that can be placed in front or back when performing the multiplications. By summing from $i = 1$ to $i = n$ and combining with Eqs. (5.4.11) and (5.4.12), the following is obtained:

$$\Sigma \hat{e}^2 = \Sigma yy' - 2\hat{b}'\bar{B} + \hat{b}'\bar{A}\hat{b} \quad (5.4.18)$$

$$= \Sigma yy' - \hat{b}'\bar{B} \quad (5.4.19)$$

$$= \Sigma yy' - (\hat{b}'\bar{B})'$$

$$= \Sigma yy' - \bar{B}'\hat{b}$$

$$= \Sigma yy' - \bar{B}'(\bar{A}^{-1} \cdot \bar{B})$$

$(\hat{b}'\bar{B})$ is a (1×1) matrix and is therefore equal to its transpose. Furthermore, $(\hat{b}')' = \hat{b}$.

If $\hat{C}$ stands for $\Sigma yy'$ and $n - p$ is the number of degrees of freedom, then the standard deviation σ will be

$$\sigma = \frac{\sqrt{\Sigma \hat{e}^2}}{n - p} = \frac{\sqrt{\hat{C} - \bar{B}'(\bar{A}^{-1}\bar{B})}}{n - p} \quad (5.4.20)$$

To get good results or a good mathematical model the number of results from the experiments has to be essentially larger than the number of regression coefficients.

The calculations of the regression coefficients b_1, b_2, b_3, . . . , b_p and the standard deviation have to be carried out by computer, perhaps by using a standard program. The regression equation established can then be used for optimization of the design with regard to the demand for power.

An example of the establishment and the use of a regression equation can be found in the papers "A Statistical Analyses of FAO Resistance Data for

Fishing Craft'' (Doust et al., 1967) and ''New Possibilities for Improvement in the Design of Fishing Vessels'' (Traung et al., 1967). The final regression equation is here expressed in the following way:

$$
\begin{aligned}
C_{R_{16}} = a_0 &+ a_1X_1 + a_2X_2 + a_3X_3 + a_4X_4 \\
&+ a_5X_5 + a_6X_6 + a_7X_7 + a_8X_8 + a_9X_9 \\
&+ a_{10}X_1^2 + a_{11}X_2^2 + a_{12}X_3^2 + a_{13}X_4^2 \\
&+ a_{14}X_5^2 + a_{15}X_6^2 + a_{16}X_7^2 + a_{17}X_8^2 \\
&+ a_{18}X_1^3 + a_{19}X_2^3 + a_{20}X_3^3 + a_{21}X_4^3 \\
&+ a_{22}X_5^3 + a_{23}X_6^3 + a_{24}X_7^3 + a_{25}X_8^3 \\
&+ a_{26}X_1^4 + a_{27}X_2^4 + a_{28}X_3^4 + a_{29}X_4^4 \\
&+ a_{30}X_6^4 + a_{31}X_2X_3 + a_{32}X_2^2X_3 + a_{33}X_2X_3^2 \\
&+ a_{34}X_2^3X_3 + a_{35}X_2^2X_3^2 + a_{36}X_2X_3^3 \\
&+ a_{37}X_1X_6 + a_{38}X_1^2X_6 + a_{39}X_1X_6^2
\end{aligned}
$$

$$
\begin{aligned}
&+ a_{40}X_1^3X_6 + a_{41}X_1^2X_6^2 + a_{42}X_1X_6^3 \\
&+ a_{43}X_1X_4 + a_{44}X_1^2X_4 + a_{45}X_1X_4^2 \\
&+ a_{46}X_1^3X_4 + a_{47}X_1^2X_4^2 + a_{48}X_1X_4^3 \\
&+ a_{49}X_2X_4 + a_{50}X_2^2X_4 + a_{51}X_2X_4^2 \\
&+ a_{52}X_4X_5 + a_{53}X_4^2X_5 + a_{54}X_4X_5^2 \\
&+ a_{55}X_4X_6 + a_{56}X_4^2X_6 + a_{57}X_4X_6^2 \\
&+ a_{58}X_4X_7 + a_{59}X_4^2X_7 + a_{60}X_4X_7^2 \\
&+ a_{61}X_4X_8 + a_{62}X_4^2X_8 + a_{63}X_4X_8^2 \\
&+ a_{64}X_1X_3 + a_{65}X_1^2X_3 + a_{66}X_1X_3^2 \\
&+ a_{67}X_2X_6 + a_{68}X_2^2X_6 + a_{69}X_2X_6^2 \\
&+ a_{70}X_5X_6 + a_{71}X_5^2X_6 + a_{72}X_5X_6^2 \\
&+ a_{73}X_1X_8 + a_{74}X_1^2X_8 + a_{75}X_1X_8^2 \\
&+ a_{76}X_2X_8 + a_{77}X_2^2X_8 + a_{78}X_2X_8^2 \\
&+ a_{79}X_5X_8 + a_{80}X_5^2X_8 + a_{81}X_5X_8^2 \\
&+ a_{82}(B_1n) + a_{83}(B_1n)^2 + a_{84}\delta_1 + a_{85}\delta_2
\end{aligned}
$$

where $C_{R(L)} = RL/\Delta V^2$, a resistance criterion that at constant Froude number enables comparison of performance to be made in terms of resistance per ton displacement

$C_{R_{16}}$ = the resistance criterion when L = 16 ft (4.9 m)

$X_1 = L/B$

$X_2 = B/T$

$X_3 = \beta$

$X_4 = \varphi$

$X_5 = \text{LCB}$

$X_6 = \frac{1}{2}\alpha_e^0$

$X_7 = \frac{1}{2}\alpha_r^0$

$X_8 = \alpha_{BS}^0$

$X_9 = \text{trim}$

B_1 -- a speed correction

n = the slope of the resistance speed curve giving the effect of tank blockage on the measured resistance of the models

$$\delta_1 = \begin{cases} 0, & \text{if there is no wooden keel} \\ 1, & \text{if there is a wooden keel} \end{cases}$$

$$\delta_2 = \begin{cases} 0, & \text{if turbulence stimulators are fitted.} \\ 1, & \text{if turbulence stimulators are not fitted.} \end{cases}$$

$a_0, a_1, a_2, \ldots, a_{85}$ are constants determined by the least-squares fitting, a different set for each value of $V/\sqrt{L}$.

Again it must be pointed out that to make a meaningful regression analysis the number of experiments has to be considerably larger than the number of constants.

Michell's integral has been used many times to calculate the wave resistance, but it must be pointed out that these theoretical methods are based on the thin ship theory of wave resistance. The methods can therefore be unacceptable from a practical point of view and, in many cases, they yield estimates wrong by as much as 100%.

The improving of method A as well as method B has elucidated many wave-resistance problems. The wave-pattern analyses have led to the discovery of the additional drag component associated with wavebreaking and to a better understanding of the effect of the bulbous bow of full large ships at low Froude number. For such ships the bulbous bow is effective in reducing the magnitude of the bow wave and thereby in avoiding wavebreaking. In high-speed vessels a bulbous bow promotes beneficial interference between waves generated at different points along the length of the hull. Thus the bulb reduces the wave resistance, and this reduction can often be predicted by use of one of the theoretical methods.