CHAPTER 7

Strength of Propellers

7.1 Introduction

The propeller is a vital component essential to the safe operation of a ship at sea. It is therefore important to ensure that ship propellers have adequate strength to withstand the forces that act upon them. On the other hand, providing excessive strength would result in heavier propellers with thicker blades than necessary, leading to a reduction in propeller efficiency. A method is therefore needed to calculate the forces acting on a propeller and the resulting stresses, so that the propeller has just the necessary strength for safe operation in service.

The forces that act on a propeller blade arise from the thrust and torque of the propeller and the centrifugal force on each blade caused by its revolution around the axis. Owing to the somewhat complex shape of propeller blades, the accurate calculation of the stresses resulting from these forces is extremely difficult. Moreover, while one may be able to estimate the thrust and torque of a propeller with reasonable accuracy for a ship moving ahead at a steady speed in calm water, it is difficult to determine the loading on a propeller when a ship oscillates violently in a seaway and the propeller emerges out of water and then plunges sharply into it at irregular intervals. The effects of the manoeuvring of a ship on the forces acting on the propeller are also difficult to estimate, particularly for extreme manoeuvres such as "crash stops". One must also take into account the fact that even in calm water the forces acting on the propeller blades are not constant but vary during each revolution due to the non-uniform wake in which a propeller works. Finally, a propeller must also withstand the effects of the stresses that may be locked into it during its manufacture, of propeller blade vibration and of corrosion and erosion during its service life.

It is thus evident that the accurate determination of propeller strength is an extremely complex problem. In practice, therefore, it is usual to adopt fairly simple procedures based on a number of assumptions to make the problem less intractable, and to allow for the simplifications by ensuring that the nominal stresses determined by these procedures have values which experience has shown to be satisfactory. The ratio of the ultimate tensile strength of a propeller material and the allowable stress (factor of safety or load factor) used in the simplified procedures for determining propeller blade strength is high, often lying between 10 to 20.

Among the simplifications made in the procedures for determining propeller blade strength are:

(i) Each propeller blade is assumed to be a beam cantilevered to the boss.

(ii) The bending moments due to the forces acting on the blade are assumed to act on a cylindrical section, i.e. a section at a constant radius.

(iii) The stresses in the cylindrical section are calculated on the basis of the simple theory of the bending of beams, the neutral axes of the cylindrical section being assumed to be parallel and perpendicular to the chord of the expanded section.

(iv) Only the radial distribution of the loading is considered, its distribution along the chord at each radius being ignored.

(v) Calculations are carried out only for the ship moving at constant velocity in calm water, the effects of manoeuvring, ship motions in a seaway and variable wake not being taken into account.

Further simplifications are made in some methods for estimating propeller blade strength.

7.2 Bending Moments due to Thrust and Torque

Consider a propeller with Z blades and diameter D operating at a speed of advance V_A and revolution rate n with a thrust T and a torque Q. The bending moments due to thrust and torque at the propeller blade section at a radius r_0 may then be determined.

Let dT be the thrust produced by the Z blade elements between the radii r and $r + dr$, Fig. 7.1. The bending moment due to the thrust on each element at the section r_0 is then:

$$dM_T = \frac{1}{Z}\, dT\,(r - r_0) \tag{7.1}$$

so that the bending moment at the section due to the thrust on the blade is:

$$M_T = \int_{r_0}^{R} \frac{1}{Z}\frac{dT}{dr}\,(r - r_0)\, dr \tag{7.2}$$

The thrust T and the bending moment due to thrust M_T act in a plane parallel to the propeller axis.

If dQ is the torque of the Z blade elements between r and $r + dr$, the force causing this torque on each of these elements in a plane normal to the propeller axis is $dQ/r\,Z$, the resulting bending moment at the section at radius r_0 being:

$$dM_Q = \frac{1}{r\,Z}\, dQ\,(r - r_0) \tag{7.3}$$

The bending moment due to torque is then:

$$M_Q = \int_{r_0}^{R} \frac{1}{r\,Z}\frac{dQ}{dr}\,(r - r_0)\, dr \tag{7.4}$$

and this acts in a plane normal to the propeller axis.

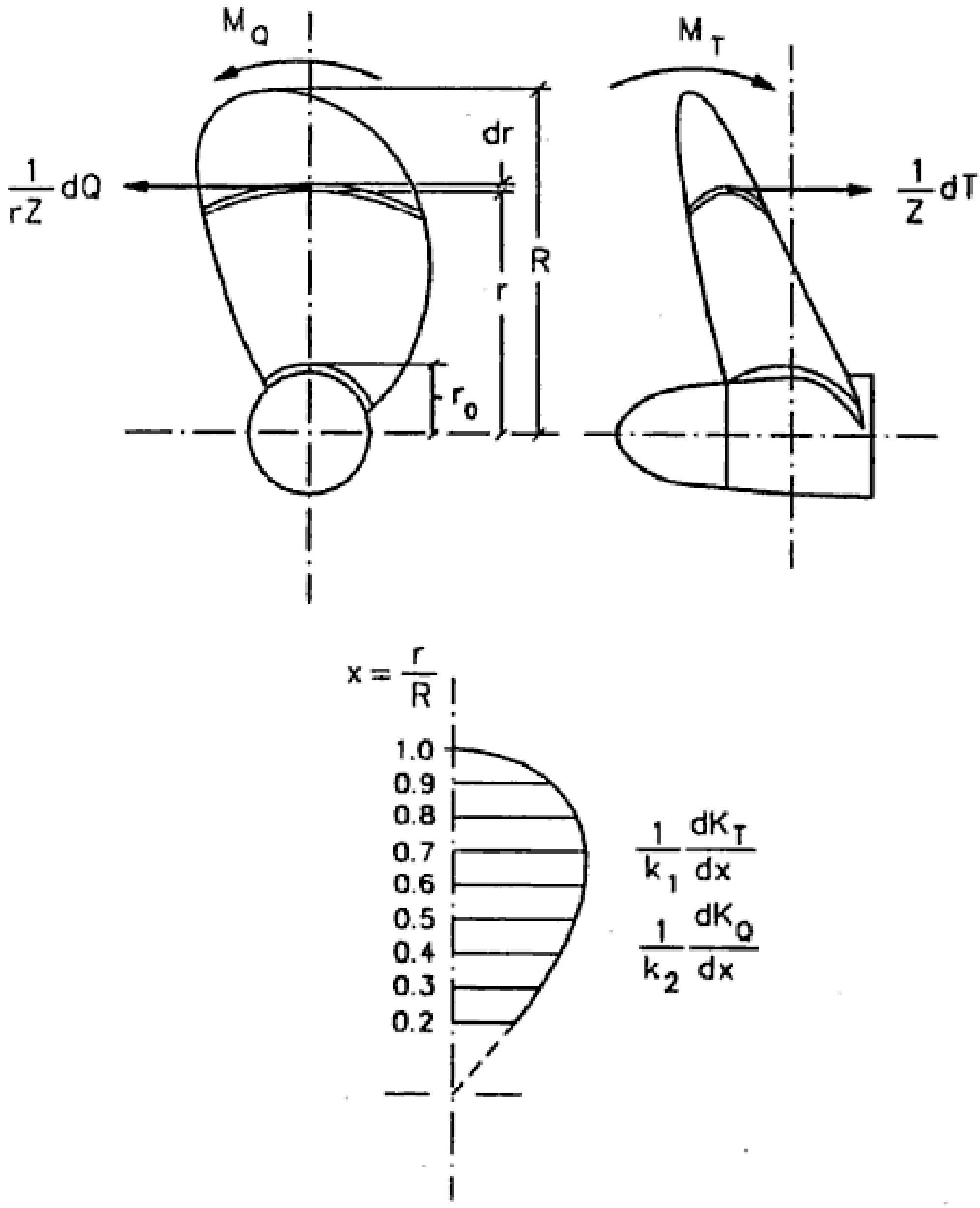

Figure 7.1 : Bending Moments due to Thrust and Torque.

Example 1

A three-bladed propeller of 3.0 m diameter has a thrust of 360 kN and a torque of 300 kN m. Determine the bending moments due to thrust and torque in the root section at 0.3 m radius, assuming that the thrust and torque are uniformly distributed between this radius and the propeller blade tip.

$$Z = 3 \qquad D = 3.0\,\mathrm{m} \qquad T = 360\,\mathrm{kN} \qquad Q = 300\,\mathrm{kN\,m}$$

$$r_0 = 0.3\,\mathrm{m} \qquad \frac{dT}{dr} \text{ and } \frac{dQ}{dr} \text{ constant.}$$

Hence:

$$T = \int_{r_0}^{R} \frac{dT}{dr}\, dr = \frac{dT}{dr}(R - r_0)$$

$$\frac{dT}{dr} = \frac{T}{R - r_0} = \frac{360}{1.50 - 0.30} = 300\,\mathrm{kN\,m^{-1}}$$

Similarly,

$$\frac{dQ}{dr} = \frac{Q}{R - r_0} = \frac{300}{1.50 - 0.30} = 250\,\mathrm{kN}$$

$$M_T = \int_{r_0}^{R} \frac{1}{Z}\frac{dT}{dr}(r - r_0)\, dr = \int_{0.30}^{1.50} \frac{1}{3}\, 300\,(r - 0.30)\, dr$$

$$= 100\left(0.5\,r^2 - 0.3\,r\right)\Big|_{0.30}^{1.50} = 72.000\,\mathrm{kN\,m}$$

$$M_Q = \int_{r_0}^{R} \frac{1}{r\,Z}\frac{dQ}{dr}(r - r_0)\, dr = \int_{0.30}^{1.50} \frac{1}{3} \times 250\left(1 - \frac{0.30}{r}\right) dr$$

$$= \frac{250}{3}\,(r - 0.3\ln r)\Big|_{0.30}^{1.50} = 59.764\,\mathrm{kN\,m}$$

It is often convenient to express the bending moments due to thrust and torque in terms of non-dimensional coefficients. Putting:

$$T = K_T\,\rho\,n^2 D^4, \qquad Q = K_Q\,\rho\,n^2 D^5, \qquad x = \frac{r}{R} \tag{7.5}$$

in Eqns. (7.2) and (7.4), one obtains:

$$M_T = \frac{\rho\,n^2 D^5}{2Z} \int_{x_0}^{1.0} \frac{dK_T}{dx}(x - x_0)\, dx \tag{7.6}$$

and

$$M_Q = \frac{\rho n^2 D^5}{Z} \int_{x_0}^{1.0} \frac{dK_Q}{dx} \frac{x - x_0}{x} \, dx \tag{7.7}$$

If $r_0 = x_0 R$ is the radius of the root section, then:

$$K_T = \int_{x_0}^{1} \frac{dK_T}{dx} \, dx \qquad K_Q = \int_{x_0}^{1} \frac{dK_Q}{dx} \, dx \tag{7.8}$$

so that:

$$M_T = \frac{K_T \rho n^2 D^5}{2Z} \frac{\int_{x_0}^{1} \frac{dK_T}{dx} (x - x_0) \, dx}{\int_{x_0}^{1} \frac{dK_T}{dx} \, dx} \tag{7.9}$$

and

$$M_Q = \frac{K_Q \rho n^2 D^5}{Z} \frac{\int_{x_0}^{1} \frac{dK_Q}{dx} \frac{x - x_0}{x} \, dx}{\int_{x_0}^{1} \frac{dK_Q}{dx} \, dx} \tag{7.10}$$

The evaluation of M_T and M_Q thus depends upon the distribution of thrust and torque over the radius. A linear distribution is sometimes assumed. However, circulation theory calculations indicate that in most propellers the thrust and torque distributions may be approximately represented by:

$$\frac{dK_T}{dx} = k_1 x^2 (1 - x)^{0.5} \tag{7.11}$$

$$\frac{dK_Q}{dx} = k_2 x^2 (1 - x)^{0.5} \tag{7.12}$$

where k_1 and k_2 are constants. Substituting these expressions into Eqns. (7.9) and (7.10), the bending moments due to thrust and torque become:

$$M_T = \frac{K_T \rho n^2 D^5}{6Z} \frac{16 - 6x_0^2 - 10x_0^3}{8 + 12x_0 + 15x_0^2} \tag{7.13}$$

and

$$M_Q = \frac{K_Q \rho n^2 D^5}{Z} \frac{8 - 2x_0 - 6x_0^2}{8 + 12x_0 + 15x_0^2} \tag{7.14}$$

In many propellers, the root section may be assumed to be at $0.2R$, so that for such propellers $x_0 = 0.2$ and:

$$M_T = 0.2376 \frac{K_T \rho n^2 D^5}{Z} = 0.2376 \frac{TD}{Z} \tag{7.15}$$

$$M_Q = 0.6691 \frac{K_Q \rho n^2 D^5}{Z} = 0.6691 \frac{Q}{Z} \tag{7.16}$$

Example 2

A three-bladed propeller of diameter 3.0 m has a thrust of 360 kN and a torque of 300 kN m at 180 rpm. The thrust and the torque may be assumed to be linearly distributed:

$$\frac{dK_T}{dx} = k_1 x \qquad \frac{dK_Q}{dx} = k_2 x$$

between the root section at $x = 0.2$ and $x = 1.0$. Determine the bending moments due to thrust and torque at the root section. How do these values compare with the values obtained by using the distributions of Eqns. (7.11) and (7.12)?

$$Z = 3 \qquad D = 3.0\,\text{m} \qquad T = 360\,\text{kN m} \qquad n = 180\,\text{rpm} = 3.0\,\text{s}^{-1}$$

$$Q = 300\,\text{kN}$$

$$\frac{dK_T}{dx} = k_1 x \qquad \frac{dK_Q}{dx} = k_2 x \qquad x_0 = 0.2$$

$$\therefore K_T = \frac{T}{\rho n^2 D^4} = \frac{360}{1.025 \times 3.0^2 \times 3.0^4} = 0.4818$$

$$K_Q = \frac{Q}{\rho n^2 D^5} = \frac{300}{1.025 \times 3.0^2 \times 3.0^5} = 0.1338$$

$$K_T = \int_{x_0}^{1.0} \frac{dK_T}{dx}\, dx = \int_{0.2}^{1.0} k_1\, x\, dx = 0.4818$$

that is:

$$k_1 \left. \frac{x^2}{2} \right|_{0.2}^{1.0} = \frac{1}{2} k_1 (1.0 - 0.04) = 0.48\, k_1 = 0.4818$$

$$k_1 = 1.00375$$

$$K_Q = \int_{x_0}^{1.0} \frac{dK_Q}{dx}\, dx = \int_{0.2}^{1.0} k_2\, x\, dx = 0.48\, k_2 = 0.1338$$

$$k_2 = 0.27875$$

$$\begin{aligned} M_T &= \frac{\rho n^2 D^5}{2Z} \int_{x_0}^{1.0} \frac{dK_T}{dx} (x - x_0)\, dx \\ &= \frac{1.025 \times 3.0^2 \times 3.0^5}{2 \times 3} \int_{0.2}^{1.0} 1.00375\, x\, (x - 0.2)\, dx \\ &= 375.0135 \left(\frac{1}{3} x^3 - 0.1\, x^2 \right)_{0.2}^{1.0} = 88.003\, \text{kN m} \end{aligned}$$

$$\begin{aligned} M_Q &= \frac{\rho n^2 D^5}{Z} \int_{x_0}^{1.0} \frac{dK_Q}{dx} \frac{x - x_0}{x}\, dx \\ &= \frac{1.025 \times 3.0^2 \times 3.0^5}{3} \int_{0.2}^{1.0} 0.27875\, x\, \frac{x - 0.2}{x}\, dx \\ &= 208.2890 \left(\frac{x^2}{2} - 0.2\, x \right)_{0.2}^{1.0} = 66.652\, \text{kN m} \end{aligned}$$

Using the thrust and torque distributions of Eqns. (7.11) and (7.12) with $x_0 = 0.2$,

$$M_T = 0.2376\,\frac{T\,D}{Z} = 0.2376\,\frac{360 \times 3}{3} = 85.536\,\text{kN m}$$

$$M_Q = 0.6691\,\frac{Q}{Z} = 0.6691\,\frac{300}{3} = 66.910\,\text{kN m}$$

(Compare these results with those of Example 1 in which uniform thrust and torque distributions have been used.)

7.3 Bending Moments due to Centrifugal Force

In addition to the bending moments due to thrust and torque, bending moments in planes parallel to the propeller axis and normal to it also arise due to the centrifugal force on each blade. If a is the area of the blade section at radius r, the mass of the propeller blade between a radius r_0 and the blade tip is given by:

$$m_b = \int_{r_0}^{R} \rho_m\, a\, dr \tag{7.17}$$

where ρ_m is the density of the propeller material. The centroid of the propeller blade will be at a radius :

$$\bar{r} = \frac{\displaystyle\int_{r_0}^{R} a\, r\, dr}{\displaystyle\int_{r_0}^{R} a\, dr} \tag{7.18}$$

so that the centrifugal force on the blade will be:

$$F_C = m_b\,\bar{r}\,(2\pi\, n)^2 = (2\pi\, n)^2\, \rho_m \int_{r_0}^{R} a\, r\, dr \tag{7.19}$$

If the distances between the centroid C of the blade and the centroid C_0 of the blade section at radius r_0 are measured as shown in Fig. 7.2, the bending

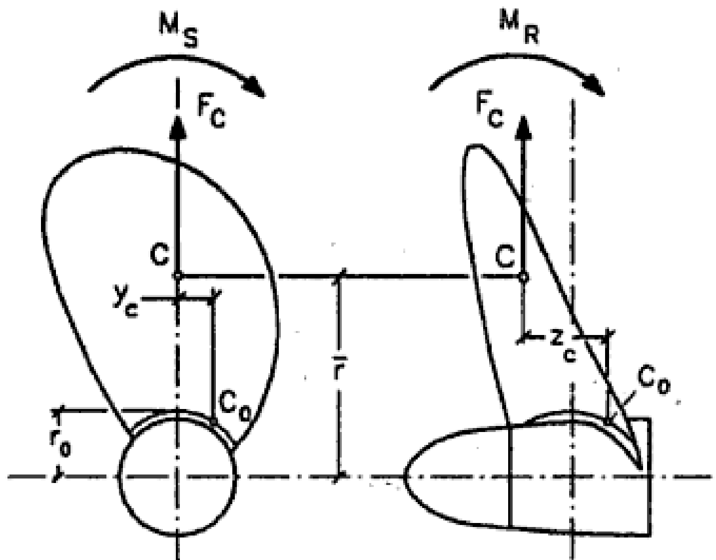

Figure 7.2 : Bending Moments due to Certifugal Force.

moments due to the centrifugal force in planes through the propeller axis and normal to it are respectively:

$$M_R = F_C \cdot z_c \tag{7.20}$$

$$M_S = F_C \cdot y_c \tag{7.21}$$

The bending moment M_R arises due to the rake of the propeller blades and acts in the same direction as the bending moment due to the propeller thrust M_T in propellers with blades raked aft. If the blades were raked forward so that the line of action of the centrifugal force passed through the centroid of the section at radius r_0, i.e. if $z_c = 0$, the bending moment due to centrifugal force in a plane through the propeller axis would be zero. The bending moment M_S arises from the skew of the propeller blades and acts in a direction opposite to the bending moment due to the torque M_Q in propellers with skewed back blades. In propellers with moderate skew, the bending moment due to skew is small and may be neglected, particularly since the error due to this overestimates the resulting bending moments and yields conservative stress values. Moreover, the existence of a bending moment due to skew contradicts the assumption made earlier that the distribution of loading across the blade is ignored. In propellers with heavily skewed blades such an assumption is obviously untenable.

Example 3

The areas of blade sections at various radii of a propeller of 3.0 m diameter are as follows:

r/R	:	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Area, m^2	:	0.0651	0.0802	0.0843	0.0807	0.0691	0.0538	0.0358	0.0168	0

The propeller runs at 180 rpm. The propeller is made of Manganese Bronze with a density of 8300 kg per m^3. Determine the centrifugal force on the blade if the root section is at $0.2R$. If the centroid of the section is at distances of 0.150 m and 0.035 m from the line of action of the centrifugal force measured parallel and perpendicular to the propeller axis, determine the bending moments due to rake and skew.

$$D = 3.0\,\mathrm{m} \qquad n = 180\,\mathrm{rpm} = 3.0\,\mathrm{s}^{-1} \qquad \rho_m = 8300\,\mathrm{kg\,m^{-3}}$$

$$x_0 = 0.2 \qquad z_c = 0.150\,\mathrm{m} \qquad y_c = 0.035\,\mathrm{m}$$

$$m_b = \int_{r_0}^{R} \rho_m\, a\, dr \qquad \bar{r} = \frac{\int_{r_0}^{R} a\, r\, dr}{\int_{r_0}^{R} a\, dr}$$

m_b and $\bar{r}$ are calculated using Simpson's Rule as follows:

x	a	SM	$f(m_b)$	$f(m_b)\,\bar{r}$
0.2	0.0651	1	0.0651	0.01302
0.3	0.0802	4	0.3208	0.09624
0.4	0.0843	2	0.1686	0.06744
0.5	0.0807	4	0.3228	0.16140
0.6	0.0691	2	0.1382	0.08292

x	a	SM	$f(m_b)$	$f(m_b)\,\bar{r}$
0.7	0.0538	4	0.2152	0.15064
0.8	0.0358	2	0.0716	0.05728
0.9	0.0168	4	0.0672	0.06048
1.0	0	1	0	0
			1.3695	0.68942

$$\int_{r_0}^{R} a\;dr \;=\; \frac{1}{3} \times \frac{1.50}{10} \times 1.3695 \;=\; 0.068475\,\mathrm{m}^3$$

$$\int_{r_0}^{R} a\,r\;dr \;=\; \frac{1}{3} \times \frac{1.50}{10} \times 0.68942 \times 1.50 \;=\; 0.0517065\,\mathrm{m}^4$$

$$\bar{r} \;=\; \frac{\int_{r_0}^{R} a\,r\;dr}{\int_{r_0}^{R} a\;dr} \;=\; \frac{0.0517065}{0.068475} \;=\; 0.755\,\mathrm{m}$$

$$m_b \;=\; \rho_m \int_{r_0}^{R} a\;dr \;=\; 8300 \times 0.068475 \;=\; 568.34\,\mathrm{kg}$$

$$F_C \;=\; m_b\,\bar{r}\,(2\pi\,n)^2 \;=\; 568.34 \times 0.755 \times (2\pi \times 3)^2\,\mathrm{kg\,m\,s^{-2}}$$

$$=\; 152.461\,\mathrm{kN}$$

$$M_R \;=\; F_C\,z_c \;=\; 152.461 \times 0.150 \;=\; 22.869\,\mathrm{kN\,m}$$

$$M_S \;=\; F_C\,y_c \;=\; 152.461 \times 0.035 \;=\; 5.336\,\mathrm{kN\,m}$$

7.4 Stresses in a Blade Section

The bending moments on the blade section at radius r_0 due to thrust and torque and those due to centrifugal force, illustrated in Figs. 7.1 and 7.2, are shown in Fig. 7.3 with reference to the blade section and its principal axes (x_0- and y_0- axes). The components of the resultant bending moment along the principal axes are then:

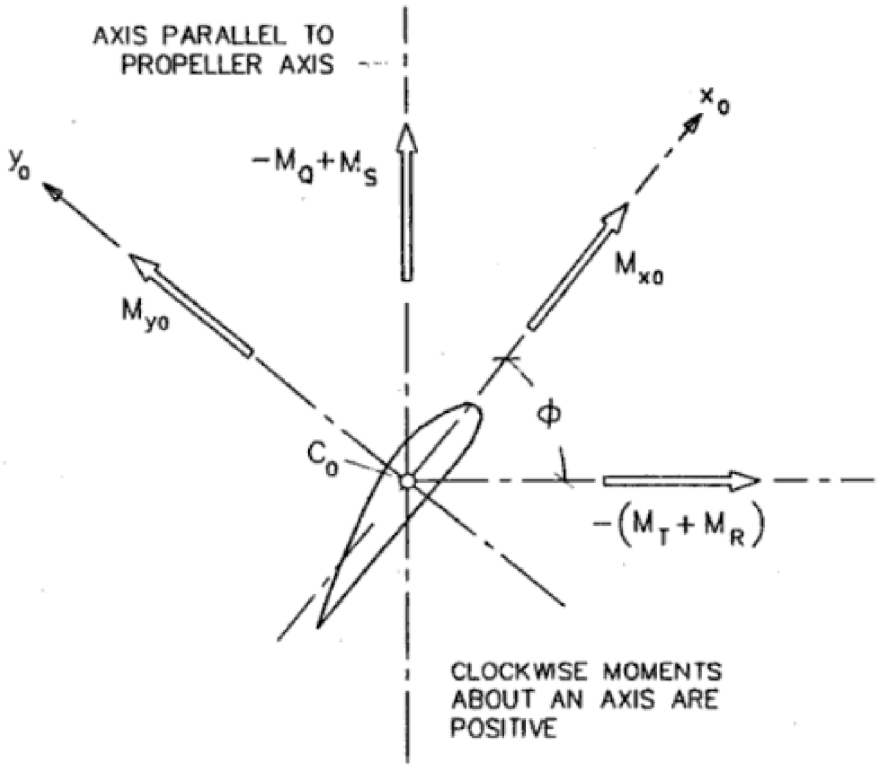

Figure 7.3 : Bending Moments at a Blade Section.

$$M_{x0} = -(M_T + M_R)\cos\varphi - M_Q \sin\varphi \tag{7.22}$$

$$M_{y0} = (M_T + M_R)\sin\varphi - M_Q \cos\varphi \tag{7.23}$$

in which φ is the pitch angle of the blade section, and the bending moment due to skew has been neglected.

If I_{x0} and I_{y0} are the moments of inertia (second moments of area) of the blade section about the x_0- and y_0- axes, and a_0 the area of the section, one may determine the stress due to the bending moment and the direct tensile stress due to the centrifugal force at any point of the section whose coordinates are (x_0, y_0):

$$S = \frac{M_{x0}}{I_{x0}/y_0} - \frac{M_{y0}}{I_{y0}/x_0} + \frac{F_C}{a_0} \tag{7.24}$$

a positive stress indicating tension and a negative stress indicating compression. It is usual to calculate the stresses at the leading and trailing edges and at the face and back at the position of maximum thickness of the blade section. For sections of normal aerofoil shape the maximum tensile and compressive stresses occur at the face and the back respectively, close to the position of maximum thickness of the section. The maximum tensile stress in the root section due to bending is then equal to the bending moment M_{x0} divided by the section modulus I_{x0}/y_0 where y_0 is the distance of the centroid from the face chord.

Example 4

A propeller of 3.0 m diameter and constant face pitch ratio 1.0 runs at 180 rpm. The bending moments due to thrust and torque are respectively 65.700 kN m and 59.800 kN m. The mass of each blade is 570 kg, the centroid being at a radius of 0.755 m. The centroid of the root section at $0.2R$ is 0.150 m forward of the centroid of the blade and 0.035 m towards the leading edge from it. The root section has a chord of 0.800 m, a thickness of 0.160 m and an area of 0.0900 m^2. The position of maximum thickness is 0.270 m from the leading edge. The centroid of the section is 0.065 m from the face and 0.290 m from the leading edge. The leading and trailing edges at the root section have offsets of 0.020 m and 0.010 m from the face chord. The moments of inertia of the section about axes through its centroid and parallel and perpendicular to the face chord are respectively 1.5×10^{-4} m^4 and 3.2×10^{-3} m^4. Determine the stresses at the leading and trailing edges, and at the face and the back.

$$D = 3.0\,\text{m} \qquad \frac{P}{D} = 1.0 \qquad n = 180\,\text{rpm} = 3.0\,\text{s}^{-1}$$

$$M_T = 65.700\,\text{kN m} \qquad M_Q = 59.800\,\text{kN m} \qquad m_b = 570\,\text{kg} \qquad \bar{r} = 0.755\,\text{m}$$

$$z_c = 0.150\,\text{m} \qquad y_c = 0.035\,\text{m} \qquad a_0 = 0.0900\,\text{m}^2$$

$$I_{x0} = 1.5 \times 10^{-4}\,\text{m}^4 \qquad I_{y0} = 3.2 \times 10^{-3}\,\text{m}^4$$

Coordinates with respect to the given axes:

Face: $x_0 = 0.290 - 0.270 = 0.020\,\text{m} \qquad y_0 = -0.065\,\text{m}$

Back: $x_0 = 0.020\,\text{m} \qquad y_0 = 0.160 - 0.065 = 0.095\,\text{m}$

Leading Edge: $x_0 = 0.290\,\text{m} \qquad y_0 = 0.020 - 0.065 = -0.045\,\text{m}$

Trailing Edge: $x_0 = -0.800 + 0.290 = -0.510\,\text{m}$

$y_0 = 0.010 - 0.065 = -0.055\,\text{m}$

$$F_C = m_b \bar{r}\,(2\pi n)^2 = 570 \times 0.755 \times (2\pi \times 3.0)^2\ \text{kg m s}^{-2}$$

$$= 152.906\,\text{kN}$$

$$M_R = F_C z_c = 152.906 \times 0.150 = 22.936\,\text{kN m}$$

$$M_S = F_C y_c = 152.906 \times 0.035 = 5.352\,\text{kN m}$$

$$\tan\varphi = \frac{P/D}{\pi x} = \frac{1.000}{\pi \times 0.2} = 1.5915 \qquad \varphi = 57.858\,\text{deg}$$

$$M_{x0} = -(M_T + M_R)\cos\varphi - (M_Q - M_S)\sin\varphi$$

$$= -(65.700 + 22.936)\,0.5320 - (59.800 - 5.352)\,0.8467$$

$$= -93.255\,\text{kN m}$$

$$M_{y0} = (M_T + M_R)\sin\varphi - (M_Q - M_S)\cos\varphi$$

$$= (65.700 + 22.936)\,0.8467 - (59.800 - 5.352)\,0.5320$$

$$= 46.082\,\text{kN m}$$

Stress:

$$S = \frac{M_{x0}}{I_{x0}/y_0} - \frac{M_{y0}}{I_{y0}/x_0} + \frac{F_C}{a_0}$$

At the leading edge:

$$S = \frac{-93.255}{1.5 \times 10^{-4}/(-0.045)} - \frac{46.082}{3.2 \times 10^{-3}/0.290} + \frac{152.906}{0.0900}\,\mathrm{kN\,m^{-2}}$$
$$= 2.5499 \times 10^{4}\,\mathrm{kN\,m^{-2}} = 25.499\,\mathrm{N\,mm^{-2}}$$

Similarly, stresses at the other points are obtained as:

Trailing edge	$S = 43.236\,\mathrm{N\,mm^{-2}}$
Face	$S = 41.821\,\mathrm{N\,mm^{-2}}$
Back	$S = -57.651\,\mathrm{N\,mm^{-2}}$

7.5 Approximate Methods

Owing to the comparative complexity of the method to determine propeller blade stresses discussed in the preceding sections, various approximate methods have been proposed. Such methods have been found to give satisfactory results and are sometimes used in the preliminary stages of propeller design when all the design details are not known. Two such methods are considered here.

A widely used approximate method is due to Admiral D. W. Taylor (1933), who considered the problem of propeller blade strength in great detail but by making various assumptions succeeded in reducing the problem to a few formulas for estimating the maximum compressive and tensile stresses in the root section of the propeller blade. The major assumptions in Taylor's method in addition to those given in Sec. 7.1 are:

(i) The thrust distribution along the propeller radius is linear.

(ii) The maximum thickness of the blade also varies linearly with radius.

(iii) The root section is at $0.2R$.

(iv) The propeller efficiency is a linear function of the apparent slip in the normal operating condition.

Based on these assumptions, the maximum compressive and tensile stresses in the root section due to thrust and torque are given by formulas, which can be put into the following form:

$$S_C = \frac{C_0 \, P_D}{Z \, n \, D^3 \, \frac{c}{D} \left(\frac{t_0}{D}\right)^2} \tag{7.25}$$

$$S_T = S_C \left(0.666 + C_1 \, \frac{t}{c}\right) \tag{7.26}$$

The additional compressive and tensile stresses due to centrifugal force are given by:

$$S'_C = C_2 \, \rho_m \, n^2 \, D^2 \left[\frac{C_3 \, \tan \varepsilon}{2 \, \frac{t_0}{D}} - 1\right] \tag{7.27}$$

$$S'_T = C_2 \, \rho_m \, n^2 \, D^2 \left[\frac{C_3 \, \tan \varepsilon}{3 \, \frac{t_0}{D}} + \frac{C_4 \, \tan \varepsilon}{\frac{c_{max}}{D}} + 1\right] \tag{7.28}$$

where:

C_0, C_1, C_2, C_3, C_4 = coefficients dependent on the pitch ratio P/D

P_D = delivered power

n = revolution rate

Z = number of blades

D = diameter

$\frac{c}{D}$ = chord-diameter ratio of the root section

$\frac{t_0}{D}$ = blade thickness fraction

$$\frac{t}{c} = \text{thickness-chord ratio of the root section}$$

$$\rho_m = \text{density of propeller material}$$

$$\varepsilon = \text{rake angle}$$

$$\frac{c_{max}}{D} = \text{maximum chord-diameter ratio of the propeller.}$$

Equations (7.25)—(7.28) are dimensionally homogeneous. However, the values of the coefficients in Table 7.1 give the stresses in kN per m^2 if P_D is in kW, n in revolutions per sec, D in m and ρ_m in kg per m^3.

Table 7.1
Coefficients for Taylor's Method

$\frac{P}{D}$	C_0	C_1	C_2	C_3	C_4
0.600	7.499	0.650	0.002568	2.750	1.590
0.700	6.471	0.710	0.002568	2.600	1.690
0.800	5.659	0.754	0.002568	2.400	1.790
0.900	5.073	0.784	0.002568	2.200	1.870
1.000	4.583	0.804	0.002568	2.070	1.925
1.100	4.190	0.817	0.002568	1.920	1.980
1.200	3.895	0.823	0.002568	1.800	2.020
1.300	3.674	0.820	0.002568	1.690	2.050

Taylor's method has been found to give satisfactory results for propellers with normal blade outlines and moderate blade area ratios. For very large blade area ratios the method gives stress values which are 10–15 percent lower than values obtained by more accurate methods.

Another approximate method for estimating propeller blade stress is due to Burrill (1959). In this method it is assumed that the thrust distribution is such that the thrust on each blade can be taken to act at a point whose distance from the root section is 0.6 times the length of the blade from root to tip. The transverse force on each blade which gives rise to the torque is similarly taken to act at a distance from the root section of 0.55 times the length of the blade. The thrust and torque bending moments can therefore be written as:

$$M_T = \frac{T}{Z} \times 0.60\,(R - r_0) = \frac{T\,D}{Z} \times 0.30\,(1 - x_0) \tag{7.29}$$

$$M_Q = \frac{Q \times 0.55\,(R - r_0)}{Z[0.55\,(R - r_0) + r_0]} = \frac{Q \times 0.55\,(1 - x_0)}{Z\,(0.55 + 0.45\,x_0)} \tag{7.30}$$

where $r_0 = x_0\,R$ is the radius of the root section.

The mass of each blade is approximated by:

$$m_b = \rho_m\,\bar{t}\,\frac{A_D}{Z}\,k \tag{7.31}$$

where $\bar{t}$ is the mean thickness of the blade from root to tip, A_D the developed blade area and k a coefficient. For a linear distribution of thickness,

$$\bar{t} = 0.50\left[(1 - x_0)\frac{t_0}{D} + (1 + x_0)\frac{t_1}{D}\right] D \tag{7.32}$$

where t_0/D is the blade thickness fraction of the propeller and t_1 is the blade thickness at the tip. A value of $k = 0.75$ is often used.

The distance of the centroid of the blade from the root section is taken as 0.32 times the length of the blade from root to tip for blades with normal outlines and 0.38 times the blade length for blades with wide tips, i.e.

$$\bar{x}\,R = x_0\,R + k_1\,(1 - x_0)\,R \tag{7.33}$$

where $\bar{r} = \bar{x}\,R$ is the radius of the blade centroid, $k_1 = 0.32$ for normal blade outlines and $k_1 = 0.38$ for wide tipped outlines.

The centrifugal force is then given by:

$$F_C = m_b\,(2\pi n)^2\,\bar{x}\,R \tag{7.34}$$

The bending moment due to rake is:

$$M_R = F_C\,(\bar{x} - x_0)\,R\,\tan\varepsilon_E \tag{7.35}$$

where ε_E is the effective rake angle, about 6 degrees greater than the geometric rake angle. The effect of skew is neglected.

The cross-sectional area of the root section and its section modulus are estimated as follows:

$$a = k_2\,c\,t \tag{7.36}$$

$$\frac{I}{y} = k_3\,c\,t^2 \tag{7.37}$$

where c and t are the chord and thickness of the root section, and k_2 and k_3 are coefficients whose values are as follows:

Section Shape	k_2	k_3
Segmental	0.667	0.112
Aerofoil	0.725	0.100
Lenticular	0.667	0.083

The stress in the root section is then given by:

$$S = \frac{(M_T + M_R)\,\cos\varphi + M_Q\,\sin\varphi}{\frac{I}{y}} + \frac{F_C}{a} \tag{7.38}$$

where φ is the pitch angle of the root section. Burrill's method gives the stress on the face of the root section at the position of maximum thickness. The maximum tensile stress normally occurs at this point.

Example 5

(a) A four-bladed propeller of 5.0 m diameter has a constant pitch ratio of 0.950, an expanded blade area ratio of 0.550 and a blade thickness fraction of 0.045. The root section is at $0.2R$ and has a chord-diameter ratio of 0.229 and a thickness-chord ratio of 0.160. The maximum chord-diameter ratio of the blade is 0.301. The propeller blades have a rake of 10 degrees aft. The propeller is made of Nickel Aluminium Bronze, which has a density of 7600 kg per m^3. The propeller has a delivered power of 5000 kW at 120 rpm. Determine the propeller blade stresses by Taylor's method.

(b) Determine the propeller blade stress by Burrill's method given the following additional data: speed of advance 7.0 m per sec, propeller efficiency 0.690 and blade thickness at tip 17.5 mm. The propeller blades have a normal outline and aerofoil sections.

(a) Taylor's method:

$$Z = 4 \quad D = 5.0\,\text{m} \quad \frac{P}{D} = 0.950 \quad \frac{A_E}{A_O} = 0.550 \quad \frac{t_0}{D} = 0.045$$

$$x_0 = 0.2 \quad \frac{c}{D} = 0.229 \quad \frac{t}{c} = 0.160 \quad \frac{c_{\max}}{D} = 0.301$$

$$\varepsilon = 10^\circ \quad \rho_m = 7600\,\text{kg m}^{-3} \quad P_D = 5000\,\text{kW}$$

$$n = 120\,\text{rpm} = 2.0\,\text{s}^{-1}$$

From Table 7.1 for $\frac{P}{D} = 0.950$,

$$C_0 = 4.828 \qquad C_1 = 0.794 \qquad C_2 = 0.002568$$
$$C_3 = 2.135 \qquad C_4 = 1.898$$

$$S_C = \frac{C_0\,P_D}{Z\,n\,D^3\,\frac{c}{D}\left(\frac{t_0}{D}\right)^2} = \frac{4.828 \times 5000}{4 \times 2.0 \times 5.0^3 \times 0.229 \times (0.045)^2}$$

$$= 52057\,\text{kN m}^{-2} = 52.057\,\text{N mm}^{-2}$$

$$S_T = S_C \left(0.666 + C_1 \frac{t}{C}\right) = 52057\,(0.666 + 0.794 \times 0.160)$$

$$= 41283\,\text{kN}\,\text{m}^{-2} = 41.283\,\text{N}\,\text{mm}^{-2}$$

$$S'_C = C_2\,\rho_m\,n^2\,D^2 \left[\frac{C_3\,\tan\varepsilon}{2t_0/D} - 1\right]$$

$$= 0.002568 \times 7600 \times 2.0^2 \times 5.0^2 \left[\frac{2.135\,\tan 10^\circ}{2 \times 0.045} - 1\right]$$

$$= 6212\,\text{kN}\,\text{m}^{-2} = 6.212\,\text{N}\,\text{mm}^{-2}$$

$$S'_T = C_2\,\rho_m\,n^2\,D^2 \left[\frac{C_3\,\tan\varepsilon}{3t_0/D} + \frac{C_4\,\tan\varepsilon}{c_{\max}/D} + 1\right]$$

$$= 1951.68 \left[\frac{2.135\,\tan 10^\circ}{3 \times 0.045} + \frac{1.898\,\tan 10^\circ}{0.301} + 1\right]$$

$$= 9564\,\text{kN}\,\text{m}^{-2} = 9.564\,\text{N}\,\text{mm}^{-2}$$

Compressive stress, $S_C + S'_C = 58.269\,\text{N}\,\text{mm}^{-2}$

Tensile stress, $S_T + S'_T = 50.847\,\text{N}\,\text{mm}^{-2}$

(b) Burrill's method:

$$V_A = 7.0\,\text{m}\,\text{s}^{-1} \qquad \eta = 0.690 \qquad t_1 = 17.5\,\text{mm}$$

Normal blade outline, aerofoil sections

$$k = 0.75 \qquad k_1 = 0.32 \qquad k_2 = 0.725 \qquad k_3 = 0.100$$

$$P_T = P_D\,\eta = 5000 \times 0.690 = 3450\,\text{kW}$$

$$T = \frac{P_T}{V_A} = \frac{3450}{7.0} = 492.857\,\text{kN}$$

$$Q = \frac{P_D}{2\pi n} = \frac{5000}{2\pi \times 2.0} = 397.887\,\text{kN}\,\text{m}$$

$$M_T = \frac{T\,D}{Z} \times 0.30\,(1 - x_0) = \frac{492.857 \times 5.0}{4} \times 0.30\,(1 - 0.20)$$

$$= 147.857\,\text{kN m}$$

$$M_Q = \frac{Q}{Z} \times \frac{0.55\,(1 - x_0)}{0.55 + 0.45\,x_0} = \frac{397.887}{4} \times \frac{0.55\,(1 - 0.20)}{0.55 + 0.45 \times 0.20}$$

$$= 68.387\,\text{kN m}$$

$$\frac{t_1}{D} = \frac{17.5}{5000} = 0.0035$$

$$\bar{t} = 0.50 \left[(1 - x_0)\,\frac{t_0}{D} + (1 + x_0)\,\frac{t_1}{D} \right] D$$

$$= 0.50\,[\,(1 - 0.20)\,0.45 + (1 + 0.20)\,0.0035\,]\,5.000\,\text{m}$$

$$= 0.1005\,\text{m}$$

$$A_E = \frac{A_E}{A_O}\,\frac{\pi D^2}{4} = 0.55 \times \frac{\pi \times 5.0^2}{4} = 10.799\,\text{m}^2 \approx A_D$$

$$m_b = \rho_m\,\bar{t}\,\frac{A_D}{Z}\,k$$

$$= 7600 \times 0.1005 \times \frac{10.799}{4} \times 0.75 = 1546.584\,\text{kg}$$

$$\bar{x}\,R = [x_0 + k_1\,(1 - x_0)]\,R = [0.20 + 0.32\,(1 - 0.20)]\,2.500$$

$$= 1.140\,\text{m}$$

$$F_C = m_b\,(2\pi\,n)^2\,\bar{x}\,R = 1546.584 \times (2\pi \times 2.0)^2 \times 1.140\,\text{N}$$

$$= 278418.5\,\text{N} = 278.419\,\text{kN}$$

$$\begin{aligned} M_R &= F_C\,(\bar{x} - x_0)\,R\,\tan\varepsilon_E \\ &= 278.419\,(0.456 - 0.20) \times 2.500\,\tan(10+6)^\circ\ \mathrm{kN\,m} \\ &= 51.095\,\mathrm{kN\,m} \end{aligned}$$

$$\begin{aligned} a &= k_2\,c\,t = k_2\left(\frac{c}{D}\right)^2 \frac{t}{c}\,D^2 = 0.725 \times 0.229^2 \times 0.160 \times 5.0^2 \\ &= 0.1521\,\mathrm{m}^2 \end{aligned}$$

$$\begin{aligned} \frac{I}{y} &= k_3\,c\,t^2 = k_3\left(\frac{c}{D}\right)^3 \left(\frac{t}{c}\right)^2 D^3 = 0.100 \times 0.229^3 \times 0.160^2 \times 5.0^3 \\ &= 3.8429 \times 10^{-3}\,\mathrm{m}^3 \end{aligned}$$

$$\tan\varphi = \frac{P/D}{\pi\,x_0} = \frac{0.950}{\pi \times 0.20} = 1.5120$$

$$\varphi = 56.520^\circ \qquad \cos\varphi = 0.5516 \qquad \sin\varphi = 0.8341$$

Stress
$$\begin{aligned} S &= \frac{(M_T + M_R)\cos\varphi + M_Q\sin\varphi}{I/y} + \frac{F_C}{a} \\ &= \frac{(147.857 + 51.095)\,0.5516 + 68.387 \times 0.8341}{3.8429 \times 10^{-3}} + \frac{278.419}{0.1521} \\ &= 45230\,\mathrm{kN\,m^{-2}} = 45.230\,\mathrm{N\,mm^{-2}} \end{aligned}$$

7.6 Classification Society Requirements

Classification Societies such as the American Bureau of Shipping and the Lloyd's Register of Shipping prescribe the strength requirements that propellers must fulfil. These include requirements for the minimum blade thickness, the fitting of the propeller to the shaft, and the mechanical properties of the propeller material.

Lloyd's Register (LR), for example, specifies the minimum propeller blade thickness at $0.25R$ and $0.60R$ for solid propellers (i.e. propellers in which the blades are cast integral with the boss, unlike controllable pitch propellers). For a propeller having a skew angle less than 25 degrees, the blade thickness neglecting any increase due to fillets is given by a formula which, in LR's notation, is:

$$T = \frac{KCA}{EFULN} + 100\sqrt{\frac{3150\,MP}{EFRULN}} \tag{7.39}$$

where:

$K = \dfrac{GBD^3R^2}{675}$

G = density of the propeller material in g/cm^3

B = developed blade area ratio

D = propeller diameter in m

R = propeller rpm at maximum power

C = 1.0 for $0.25R$ and 1.6 for $0.60R$

A = rake at blade tip in mm (positive aft)

E = actual face modulus/$0.09T^2L$, but may be taken as 1.0 and 1.25 respectively for aerofoil sections with and without trailing edge washback

T = blade thickness in mm at the radius considered, i.e. $0.25R$ or $0.60R$

L = length in mm of the expanded cylindrical section at the radius considered

U = allowable stress in N per mm^2

$F = \dfrac{P_{0.25}}{D} + 0.8$ for $0.25R$

$\;\;\; = \dfrac{P_{0.6}}{D} + 4.5$ for $0.6R$

N = number of blades

Twin screw ships

$$w = 1.7643\,C_B^2 - 1.4745\,C_B + 0.2574 \tag{A4.2}$$

Schoenherr (Rossell and Chapman, 1939)

Single screw ships

$$w = 0.10 + 4.5\frac{C_{PV}\,C_P\,B/L}{(7 - 6\,C_{PV})(2.8 - 1.8\,C_P)} + 0.5\left(\frac{T-h}{T} - \frac{D}{B} - k'\varepsilon\right) \tag{A4.3}$$

$k' = 0.3$ for normal sterns

$= 0.5$-0.6 sterns with cutaway deadwood

ε in radians.

Twin screw ships

With bossings and outward turning propellers

$$w = 2\,C_B^5\,(1 - C_B) + 0.2\cos^2\tfrac{3}{2}\psi - 0.02 \tag{A4.4}$$

With bossings and inward turning propellers

$$w = 2\,C_B^5\,(1 - C_B) + 0.2\cos^2\tfrac{3}{2}(90 - \psi) - 0.02 \tag{A4.5}$$

With propellers supported by struts

$$w = 2\,C_B^5\,(1 - C_B) + 0.04 \tag{A4.6}$$

Burrill (1943)

Single screw ships

$$w_F = \frac{w}{1-w} = 0.285 - 0.417\,C_B + 0.796\,C_B^2 \tag{A4.7}$$